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Effect of Compression Ratio on NO_X Production by Gas Turbines

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IT is well established as a result of correlations by Lipfert,[1] supported by Bahr[2] and by Nelson,[3] that the mass fraction of NO_x in the exhaust gases of currently operational aircraft gas turbines increases dramatically with compressor outlet temperature, hence with compression ratio. Sawyer et al.[4] and Ferri and Agnone[5] prefer a correlation in terms of maximum combustion temperature, but for the rather narrow range of equivalence ratios in the primary zones of existing combustors these correlations appear to be essentially equivalent.

The purpose of this Note is to point out that these correlations do not necessarily imply that a higher NO_x fraction in the exhaust must be accepted as a result of increased compression ratio and turbine inlet temperature, even for currently available burner technology.

As Lipfert[1] and Sawyer et al.[4] point out, the data correlations imply near constancy of the residence times in the primary zones of the burners for the engines correlated. The dramatic rise in NO_x with increasing compressor outlet temperature then follows from an increase in the rate of NO_x formation roughly according to the relation

$$(NO_x) \propto p_3^{1/2} e^{-2400/T_3} \tau_p$$

where p_3 and T_3 are compressor outlet pressure and temperature and τ_p is the residence time in the primary zone.

Note, however, that since the rates of the combustion reactions also are increased as T_3 and p_3 increase, it should be possible to decrease τ_p as T_3 and p_3 are increased. Indeed, correlations of combustor efficiency, η_b, for fixed geometry combustors have indicated a correlation of the form[6]

$$\eta_b(p_3^{1.75} e^{T_3/b}/m)$$

where m is mass flow through the burner, and b varies from 300 K for a fuel air ratio of 0.016 to 150K for 0.010. Taking $\tau_p \propto p_3/m$, i.e., neglecting small temperature changes in the primary zone, and assuming $p_3 \propto T_3^{(\gamma-1)/\gamma}$ we find that for *constant combustion efficiency*

$$(NO_x) \propto e^{-[(2400/T_3)+(T_3/b)]} / T_3^{0.25\gamma/(\gamma-1)} \quad (\eta_b = \text{constant})$$

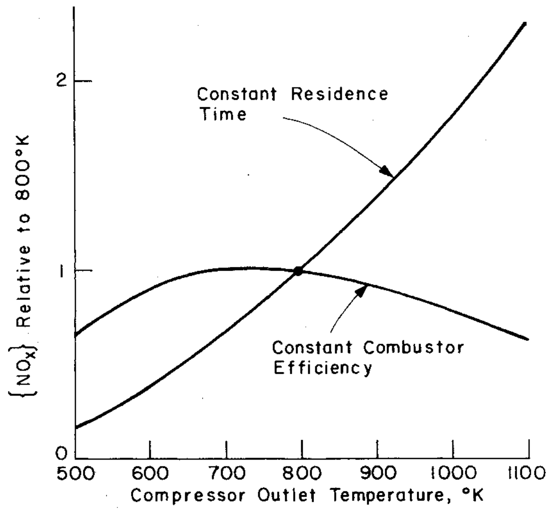

Fig. 1 Comparison of the variations of NO_x mass fraction with compressor outlet temperature which should result for burners having constant combustion efficiency, with that for burners having constant primary zone residence time.

This result is plotted in Fig. 1, compared to the corresponding result for τ_p = constant. We see that the NO_x mass fraction is sensibly constant over the range of T_3 from 600 to 900K, and is even predicted to fall off at higher compressor outlet temperatures. Hence the conclusion that for *constant combustion efficiency in a given combustor design,* NO_X *is nearly independent of combustor inlet temperature.*

The author is aware that the requirement for low CO and hydrocarbon efflux at part throttle conditions dictates high combustion efficiency at these conditions, and that combustion efficiency tends to decrease with decreased throttle setting for fixed geometry combustors. The part throttle requirements can be and have been met with fixed geometry combustors by choosing residence times longer than are required for full power operation. This solution is attractive in part because combustor length and volume are not as critical in large high pressure ratio engines as they were in early small turbojets.

It is suggested therefore, that NO_x production in high-pressure ratio engines can be reduced by designing for the minimum primary zone residence time consistent with combustion efficiency at full power, and then achieving satisfactory part throttle combustion efficiency by some scheme such as compressor outlet bleed or combustor bypass.

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Multiple Slot Skin Friction Reduction

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Introduction

ATTEMPTS by fluid dynamicists to minimize skin friction drag on aerodynamic vehicles has received considerable attention in the past. Various methods have been investigated, but associated and often frustrating penalties or hardware problems have prevented the development of functional and economical systems for full scale aircraft application. In light of increased energy costs, there is a resurgence of interest in viscous skin friction drag reduction techniques.[1] In addition, techniques for reducing aircraft form drag have increased the percentage of the total aircraft drag attributable to viscous effects.

One possible technique for reducing skin friction drag is to inject relatively low momentum air through discrete tangential slots on external surfaces of an airplane (i.e., slot injection). Although numerical and experimental studies at high speeds have shown the potential of slot injection as a skin friction reduction technique,[2-5] there is little experimental skin friction data for discrete slot injection in subsonic flows. This paucity of low-speed skin friction data with slot injection prevents a realistic systems analysis of slot injection as a viable drag reduction technique. The purpose of the present study was to investigate analytically the effect of slot injection on skin friction for a representative fuselage shape (ogive-cylinder body) and to indicate the potential of slot injection as a drag reduction system in subsonic flow. The numerical technique used[6] was selected because it predicted well the experimental slot injection skin friction data obtained at supersonic and hypersonic speeds.

Description of the Investigation

The study was conducted for typical CTOL cruise flight conditions and a fuselage shape representative of current long-haul subsonic transports. The Mach number and altitude chosen were 0.82 and 11 km, respectively. The fuselage length (L) was 67.06 m with a maximum diameter of 7.32 m. the shape of the fuselage was defined by the following equations

$$r = \frac{3}{55}\left[1 - \left(\frac{11(x)-2}{2}\right)^2\right]^{1/2} \text{ for } 0 \leq x \leq \frac{2}{11}$$

$$r = \frac{3}{55} \qquad \text{for } \frac{2}{11} \leq x \leq 1.0$$

where r and x are the local body radius and axial distance normalized by L. The surface pressure distribution up to the first slot ($x=0.09$) was determined for the given body shape from the method of Ref. 7; from the first slot to the end of the fuselage the local pressure was assumed constant and equal to the freestream static pressure. The numerical method of Ref.

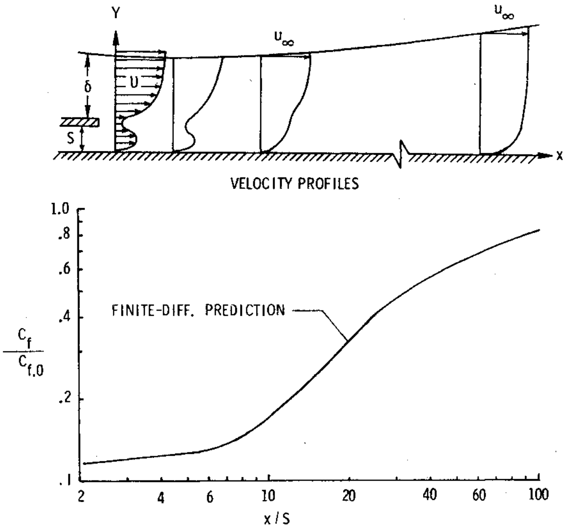

Fig. 1 Velocity development and skin friction behavior downstream of a single slot.

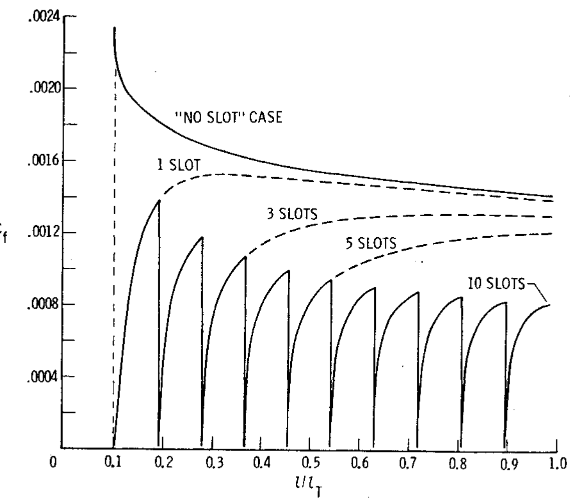

Fig. 2 Skin friction reduction with slot injection, $S=7.46$ cm $(u/u_\infty)_{max}=0.34$.

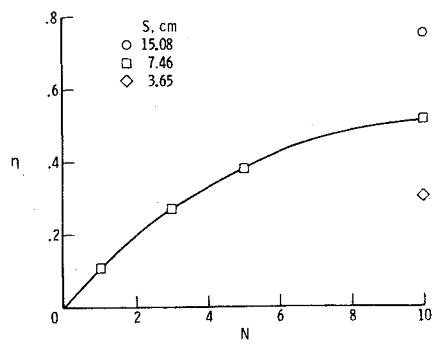

Fig. 3 Skin friction reduction effectiveness as a function of number of slots.

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